Title of the course (20 hours):
Stochastic modeling of time series, selected problems

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Short description
This course discusses some selected statistical methods of analysis of empirical time series. The main technique discussed is the Kalman filter (KF), a mathematical power tool that is playing an increasingly important role in solving different practical and scientific problems, ranging from the satellite navigation programs, estimation of the geodynamic parameters based on different data sets, up to a wide variety of computer graphics applications. The Kalman filter is the best possible estimator for a large class of problems and a very effective and useful estimator for an even larger class. An important property is its simplicity. In the linear version discussed here, the KF equations can be easily programmed and used for analysis of empirical data.

The presentation of the Kalman filter is preceded by description of the autoregressive (AR) modeling of time series. The AR models are useful in analysis of empirical data sets, example being a high-resolution spectral analysis method based on the maximum entropy criterion. What is also important, the AR models can be incorporated into the Kalman filter, greatly enhancing its applications.

A certain advantage of this course is that we allow complex parameters in the models discussed. That comes from the personal experience of the lecturer who applies the KF algorithm for the Earth rotation studies using the complex variables. Nevertheless we believe that the use of complex variables can be also useful in other fields of the KF applications.

It is assumed that participants of the course have a basic knowledge of the complex number arithmetic, the linear algebra, the theory of ordinary differential equations, and the theory of probability.

Program details:

1. Introduction
   - 1.1. A short review of the complex numbers arithmetic, fundamental theorem of algebra.
   - 1.2. A short review of probability and random processes – probability space, scalar random variables: probability distribution and density functions, parameters of random variables – expected value, variance, standard deviation, asymmetry factor and excess factor; multivariate random variables: probability distribution and density functions, independent variables, covariance and correlation coefficient, covariance matrix, multivariate normal distribution.
   - 1.3. Introduction to the theory of random processes – deterministic signals: periodis, harmonic, polyharmonic and transient signals, Fourier series and Fourier integral; random signals: stationary, ergodic and nonstationary signals; characteristics of random signals: mean square value, probability density function, autocorrelation, power spectral density, joint characteristics of random signals: joint probability density function, intercorrelation, cross power spectral density; appendix: definition and properties of *Dirac delta function*.
2. Modeling autoregressive (AR) processes
   - 2.1. Discrete AR process – definition, difference operator form, AR process as output of the linear filter; frequency domain representation: transfer function, stationarity condition; autocorrelation function: Yule-Walker equations, variance, power spectral density function, factorization of the AR-operator and explicit form of the autocorrelation function, AR prediction function, examples.

2.2. Continuous AR process – definition, differential operator form, AR process as output of the linear filter; frequency domain representation: transfer function, power spectral density function; autocorrelation function: derivation of the differential equation and its solution.
2.3. Relationship between coefficients of the continuous AR model and its discrete-time representation.
2.4. Estimation of the parameters of AR models for real data – estimation based on sample autocovariance, criteria for finding an optimal AR order, estimation by the maximum entropy method (MEM), spectral analysis based on AR model.
3. Kalman filter (KF)
3.1. Mathematical background I. elements of linear algebra - eigenvalues and eigenvectors, diagonalizing a matrix.
3.2. Mathematical background II. linear ordinary differential equations with constant coefficients - solution of the homogeneous equation by the eigenvalue approach, transition matrix, solution of the non-homogeneous equation, stability of the solution, examples.
3.3. Linear dynamical systems – state-space formulation for continuous-time variables, solution of the system and its stability, discretization.
3.4. Linear KF algorithm for discrete observations – state-space formulation, measurement model, observability condition, optimal extrapolation and update, Kalman gain matrix, KF algorithm, discussion of simple examples.
3.5. Scientific applications of the Kalman filter - optimal smoothing of experimental data, solving deconvolution problem by the use of AR models in KF, special case: application of KF for studying geophysical excitation of polar motion.

Bibliography

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